Deliberate Selection of Mathematics Standards Repeating Similar Content

Panelists selected similar content—mainly in algebra, but also in functions, number, and geometry—from Levels D (grades 6–8) and E (high school). The overlapping high school content from Level E often subsumes its Level D counterpart, but at a higher level of rigor. For example, both levels emphasize linear equations, while the high school standards extend the requirements to quadratic and exponential functions. Panelists decided to include the overlapping grade 6–8 standards because they tended to offer a more thorough description of the demands. For example, standards 8.EE.8b and A.REI.6 both require the solution of a system of linear equations, yet both were selected. While the grade 8 standard is subsumed by the high school standard, it provides descriptions of methods of finding the solution, the reasons for using those methods, and useful examples.

<table>
<tr><th colspan="3">Connections Between Level D and Level E</th></tr>
<tr><th>Level D</th><th>Level E</th><th>Comments on Connections</th></tr>
<tr><td rowspan="4">7.EE.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”</td><td>A.SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.</td><td rowspan="4">Both levels address various ways to rewrite an algebraic expression or equation and reasons for doing so.</td></tr>
<tr><td>A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</td></tr>
<tr><td>A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.</td></tr>
<tr><td>A.CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law V = IR to highlight resistance R.)</td></tr>
<tr><td>7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</td><td>A.CED.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</td><td rowspan="3">Both levels require writing equations and inequalities. Level D standards here indicate only linear equations and inequalities, while the Level E connection includes linear but also extends to quadratic, simple rational, and exponential.</td></tr>
<tr><td>7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</td><td rowspan="2">A.REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td></tr>
<tr><td>7.EE.4b Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.</td></tr>
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Connections Between Level D and Level E		
Level D	**Level E**	**Comments on Connections**
7.G.6 Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.		
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	**G.GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	Both levels are concerned with finding volumes, although the Level E standard specifies and extends the types of 3-D figures required.
7.SP.4 Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in one chapter of a science book are generally longer or shorter than the words in another chapter of a lower level science book.*	**G.MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	Both levels require problem solving using rates and ratios. Problems related to density in Level D are applications of rates and ratios as a geometric model.
8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{(-5)} = 3^{(-3)} = (1/3)^3 = 1/27$. (8.EE.1)	**S.ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	Both levels require interpretation of measures of center and spread (variability), and both imply comparison of two or more data sets. The Level E standard also adds interpretation of the shape of the data and the context in which it is found.
8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	**F.IF.8b** Use the properties of exponents to interpret expressions for exponential functions. Use properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in an exponential function and then classify it as representing exponential growth or decay.* (F.IF.8b)	Both levels address using the properties of exponents. Level D deals with generating solutions; Level E extends the standard to interpreting expressions.

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8.EE.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	**A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	These connect through the solution to radical equations. The Level E standard extends to rational equations. The domain restriction in Level D hints at a knowledge of extraneous solutions.
8.EE.7 Solve linear equations in one variable.	**N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	Both levels deal with precision and accuracy in these standards. Using the appropriate level of precision (Level E) is important to calculations with scientific notation for quantities that are very large or small (Level D).
8.EE.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*	**A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	Both Levels D and E require solving one-variable linear equations.
8.F.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	**A.REI.6** Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	These standards both require solutions for a system of linear equations, using multiple strategies.
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	**F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	Defining a function and its graph is required in both Levels D and E.

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8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	**F.BF.1** Write a function that describes a relationship between two quantities.*	This Level E standard is more generally stated than the Level D counterpart, but addresses the requirement to "construct" a function.
	F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	Both levels require interpretation of the parameters of a linear function. Level E also extends to exponential functions.
8.G.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	**A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [Also see 8.F.5]	While this Level E standard does not expressly address the "qualitative" description of the relationship, it does address the graphic analysis of that relationship.
	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	Both levels address the features of a graph, as stated in the second part of the Level D counterpart.
8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	**G.SRT.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	While both levels address congruence and similarity, they are approached somewhat differently in the levels. In the Level D standards, the emphasis is on demonstrating congruence/similarity through transformations. The Level E counterpart does not imply that transformations must be used to prove congruence/similarity.
8.SP.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.		

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8.SP.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	**S.ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).	Both levels require using scatter plots to represent data. The Level D counterpart also requires interpretation and analysis of the plots, while at Level E, other types of plots also are specified.
8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they like to cook and whether they participate actively in a sport. Is there evidence that those who like to cook also tend to play sports?*	**S.ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Both levels address interpretation of slope and intercept in models created from data.
	S.ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Analysis of frequency of data points and two-way tables to display and analyze those frequencies are required in both levels. Level D gives a detailed description of the tables, while Level E also requires interpretation in the context of the data, including recognition of associations and trends.